



RESERVE

D2.3 v1.0

Linear Swing Dynamics Validation and Application in Future Converter-Based Power Systems

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Abstract:

This deliverable presents, discusses and validates the concept of Linear Swing Dynamics (LSD) for future power systems with up to 100% generation from converter-based Renewable Energy Systems (RES). The LSD-based Improved Virtual Synchronous Generator (LSD-VSG) has been developed to take the advantages of conventional Synchronous Generators (SG), such as generation-system synchronization and inertial response, and tackle its disadvantages represented by the nonlinear characteristics and nonlinear swing dynamics. The proposed LSD-VSG has the objectives of achieving linear dynamical system with the capability of providing virtual inertia and frequency support. The LSD-VSG has been tested and validated in the Single Machine Infinite Bus (SMIB) system. Also, stability analysis has been performed to study LSD-VSG dynamics and tune the control parameters to achieve damped response and enhanced dynamic performance.

Keyword list:

Converter-based RES, frequency control, linear swing dynamics, stability analysis, synchronverter, virtual inertia, virtual synchronous generator.

Disclaimer:

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Executive Summary

This deliverable presents the development and validation of Linear Swing Dynamic (LSD) concept in future converter-based power systems, within the wider context of Work Package (WP) 2 and RESERVE. WP2 focuses on solving the challenges of stabilising system frequency WP2 focuses on the detailed analysis of the challenges and solutions for frequency control and linear swing dynamics in the new energy network architecture characterized by 100% Renewable Energy System (RES).

The future power systems will experience very high displacement of Synchronous Generators (SG) with up to 100% converter-based RES. Note that a SG is characterized by the inherent nonlinear swing dynamics due to the electromechanical oscillations. This appears in the well-known nonlinear power-angle characteristics. This nonlinearity poses critical limitations and challenges in analysing system stability, frequency control and regulation. In this regard, this deliverable proposes the concept of LSD that aims to achieve linear swing dynamics and linear power-angle characteristics. This concept is embedded in the control of RES-tied converters of future power systems with up to 100% generation from converter-based RES.

This deliverable proposes a new control scheme for RES-tied converters and it is called LSD-based Virtual Synchronous Generator (LSD-VSG). The aim is to

- Achieve linear and uniform system swing dynamics. This is due to the linear characteristics of LSD, in contrast with conventional SG
- Preserve system synchronization and coherency
- Emulate the SG and contribute to virtual inertia provision and system frequency regulation
- Regulate the voltage through LSD-based voltage control
- Possibility of capturing system stability information for both small and large disturbances

For concept validation, the LSD-VSG has been tested, under different operating conditions, in Single Machine Infinite Bus (SMIB) system. The results show that the LSD-VSG was able to fulfill the above mentioned objectives. As the LSD concept adopts a proper voltage control, which is part of LSD-VSG control structure, the realization of LSD characteristics (linearized power-angle characteristics) depends on the permitted tolerance of voltage deviation in practice. However, from a theoretical point of view, the LSD concept can be realized with no limits, i.e. with infinite operating conditions.

The proposed LSD-VSG has been simulated and tested in Single Machine Infinite Bus (SMIB) system. Also, a comparison in the power-angle characteristics of LSD-VSG and classical SG/VSG is presented under different operating conditions.

Stability analysis is performed to study LSD-VSG dynamics and operation modes. The respective control parameters have been tuned based on the conducted stability analysis and time-domain simulations. This in turn, aims to achieve better damped response, along with enhanced dynamic performance.

Concluding remarks are reported, based on the conducted analysis and obtained results. The obtained results prove that the LSD (linearised power-angle) characteristics can be achieved till a maximum operating point, on the power-angle linearized curve, which is likely the maximum operating margin in practical system operation to deliver the corresponding maximum active power. Further development in the concept and its control structure, for more sophisticated systems, is discussed in future work.

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1. Introduction

1.1 Motivation

The future power systems with up to 100% generation from converter-based (non-hydro) (converter-based) Renewable Energy System (RES) will experience radical changes in system dynamics, control and automation architecture. The displacement of classical Synchronous Generator (SG) with a converter-interfaced RES will result in a low/zero (rotational or mechanical) inertia power systems with very fast and complex dynamics. This in turn, poses critical challenges in system control, stability and dynamic performance. Hence, call for new advancements is required to analyse future system dynamics and develop innovative concepts that tackle the above challenges and facilitates a smooth transition towards well-controlled, stable and reliable power systems in future. This could be achieved by exploiting the smartness of the deployed power electronic interfaces of RES. In this regard, the concept of Linear Swing Dynamics (LSD) is developed independently from the legacy representation of a SG, and is tested and verified in this deliverable. The LSD concept is embedded in the control of RES-tied converters to achieve linear dynamical system. This is in contrast with the SG which has the main drawback of nonlinear swing characteristics due to the electromechanical oscillations, which in turn affect frequency control and system dynamic performance.

1.2 Connection with other Tasks of the Project

The research work conducted within Task 2.3 (Linear Swing Dynamics Validation) adopts the scenarios presented in WP1 (Scenarios and Architecture). The developed concepts and control schemes within this task will be tested and validated in the real-time simulation in WP4 (Simulation and Field Trials) as shown in Figure 1. Specifications and requirements on system communication will be reported as an input to Task 2.4 (Definition of ICT Requirements for Frequency Control Concepts). Eventually, conclusions and recommendations will be given, from the conducted research work and obtained results, to participate in defining new ancillary services and network codes in WP6 (Regulatory, Legal Issues and Business Models for RES).

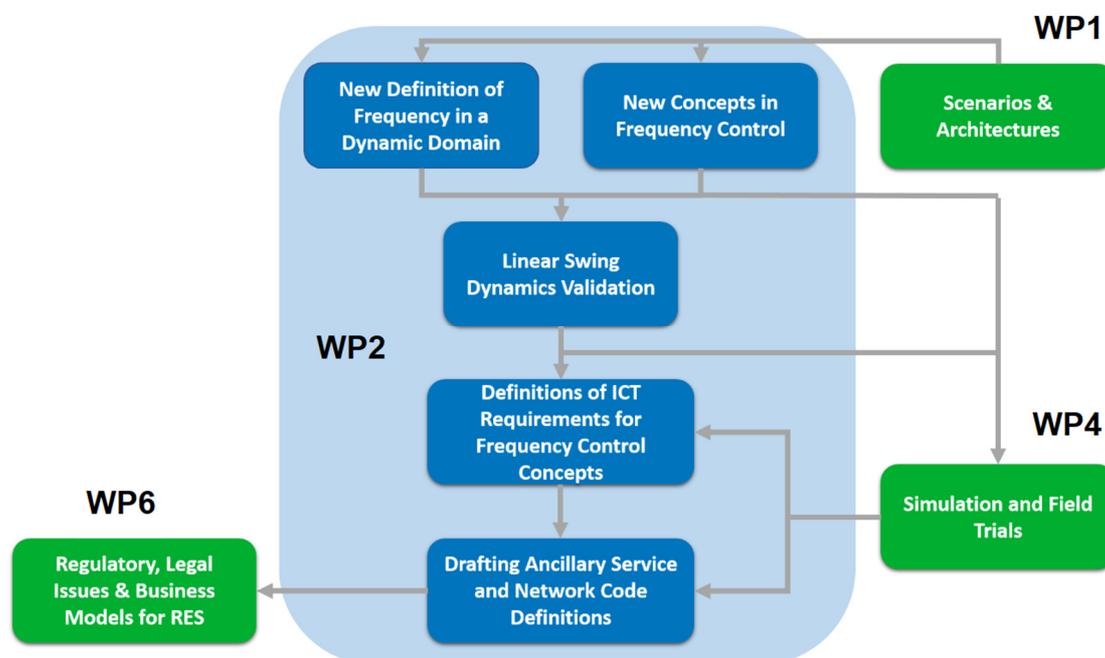


Figure 1. Tasks dependencies within WP2.

1.3 Structure of the Deliverable

This deliverable is organized as follows. Chapter 2 provides a background of power system dynamics and stability. Also, it discusses the implications of swing equation on system stability, control and dynamic performance. Chapter 3 presents the up-to-date proposed solutions for RES-tied converters. Chapter 4 explains and discusses the LSD concept in future converter-based power systems. Chapter 5 covers the development of LSD-based control for RES-tied converter (LSD-VSG), time-domain simulations, and the conducted stability analysis. Finally, Chapter 6 discusses the concluding remarks, along with the future work.

1.4 How to read this Deliverable

This document can be read independently, but the authors suggest reading the deliverables D2.1 and D2.4 in parallel.

Overall, this deliverable (D2.3) is related to the following documents from the RESERVE project:

- D1.5 Adaptation of Research Concepts based on Simulation, Live ICT Tests and Field Trial Results
- D2.1 Definition of Frequency under High Dynamic Conditions
- D2.2 Concepts of Primary and Secondary Frequency Control
- D2.4 Definition of ICT Requirements for Frequency Control

2. Power System Dynamics and Stability

2.1 Power system stability

Power system stability is the ability of an electric power system, for a given operating condition, to regain a equilibrium state after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact [1]. It is related to specific factors depending upon the nature of disturbance and the time frame. The power system stability is classified into three categories: rotor-angle stability, frequency stability, and voltage stability. It is worth mentioning that in this report, only the rotor-angle stability and frequency stability will be addressed. Voltage stability is outside the purpose of this report [1-3]. More details on power system stability are provided in the Annex A.1.

2.2 Swing equation of synchronous generator

A power system comprises multiple synchronous generators (SG) operating synchronously under different operating conditions. In case of normal operating conditions, the relative position of the rotor axis and the resulting magnetic field of each SG is fixed, and the angle between them is known as the load angle. In case of disturbances, the rotor accelerates or decelerates with respect to the synchronously rotating air gap magneto-motive force (m.m.f.), creating relative motion. The equation that expresses the relative motion is known as the swing equation. The original form of the swing equation is a non-linear second order differential equation which depicts the swing of the SG rotor [4]. The power exchange between the mechanical rotor and the electrical grid due to acceleration/deceleration of the rotor is called inertia response. More details, together with the mathematical formulation of swing equation, are provided in the Annex A.2 [5], [6].

2.3 Implications of swing equation in future system stability and control

The swing equation is adopted in power system stability analysis, frequency control and regulation. In case of load decrease in the grid, an accelerating torque will be created that increases the speed, and hence, increases system frequency. This is known as over-frequency scenario. On the other hand, in case of load increase, decelerating torque will be created, that decreases the speed of the machine and system frequency. This is known as under-frequency scenario.

As it results from the Annex A.2, in future power systems, where the SGs are replaced by converter-based RES, the system swing dynamics will have different characteristics and exhibits different behaviour.

The conventional high-inertia power systems where the frequency usually exhibits small variations in case of normal power unbalances, will no longer be valid in future systems, characterized by high share of converter-based RES. This has critical implications on frequency dynamics, power system stability and operation. Note that frequency dynamics are much faster in future low/zero inertia power systems, making frequency control and power system operation more challenging [7] and [8].

The above challenges stimulate for the call and development of innovative concepts that: interpret/analyse future system dynamics, facilitate the deployment of effective solutions, analyse flexibly the system stability, and achieve consistent control performance. In this regard, this deliverable proposes the concept of LSD, as discussed in Chapter 4.

2.4 Operation and control of low inertia power systems

With the incessant increase in on- and off-shore RES integration, in different power capacity levels, along with the available flexibility in terms of control provided by the converter-interfaced generation and load, it is not far-fetched to envision a future inertia-less scenario.

However, the concept of generation/load balance and frequency control (regulation) in electrical grids is linked to the mechanical (rotating) inertia in the system. When considering a low/zero inertia power system, a question always arises whether the system could be satisfactorily operated and controlled with either reduced or zero inertia.

It is worth mentioning that the converters will play the major role in future systems in the transition from grid supporting to grid forming. It would be imperative that large converters (with high power levels) are operated as a voltage source in a grid forming mode. This means that they have to be able to set the voltage level of the system, and other small (low power levels) converters can follow. This would require a shift in the control ideology. However, significant precaution must be taken to ascertain that the converters have to participate in system frequency and/or voltage stabilisation without violating (compromising) their operation performance.

These aspects, along with the importance of system frequency dynamics and the acceptable operating limits are yet to be carefully investigated, from system dynamics and performance perspectives point of view. In this regard, there are different solutions proposed in the literature to achieve a secure and stable integration of RES in low-inertia power systems, such as Synchronverter and Virtual Synchronous Generator, etc.

3. Call for Smart Converters for Future low Inertia Power Systems

Recently, an intensive research work has been conducted in the subject of control development of converter-interfaced RES. The aim is to emulate the classical SG in providing system synchronization and virtual inertia provision and provide frequency and voltage regulation as ancillary services. Examples of that are the Virtual Synchronous Generator (VSG) and Synchronverter (SV). The main differences between the two solutions is that the former is based on d-q coordinates control and uses a dedicated synchronization unit, e.g. Phase Locked Loop (PLL). On the other hand, the latter does use the d-q coordinates in its control and might have a self-synchronization unit. Both VSG and SV are considered in the research work being conducted within the related Task 2.3. However, this deliverable presents the development, stability analysis and validation of the improved LSD-based VSG. The development of LSD-based SV, along with a comparative study between LSD-VSG and LSD-SV will be done in future work.

3.1 Virtual synchronous generator

The VSG is proposed in literature to emulate the SG and provide virtual inertia for system frequency support [11] and [12]. It is composed of power and control parts as shown in Figure 2. The VSG control is based on d-q coordinates, and it is composed of upper control level (including virtual inertia emulation and inner current loop) and lower control level (including capacitor voltage balancing, Pulse Width Modulation (PWM), etc.). Using the park transformation, the three phase quantities (measured voltage and current) are transferred from abc to d-q coordinates. The considered measurements for the control are the active power P_{meas} , reactive power Q_{meas} , system frequency ($\omega=2\pi f$), grid side current i_o , grid voltage V_{meas} , and filter's capacitor current (i_{cv}). The measured V and i_{cv} quantities are transformed to d-q coordinates to achieve v_{dq} and $i_{cv,dq}$, respectively, which are fed as an input to the control. The variables P , Q , and V are controlled based on their reference magnitudes P_{ref} , Q_{ref} , and V_{ref} .

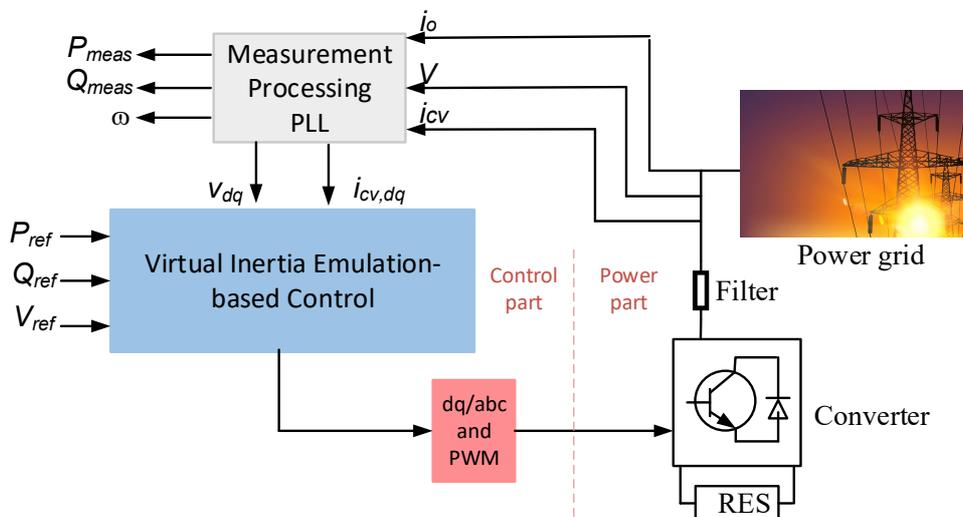


Figure 2. Generic scheme of Virtual Synchronous Generator

A brief description of the VSG control is provided as follows. A nested control loop structure is adopted where current and voltage can be controlled separately. The control strategy is implemented in the synchronously rotating reference frame, based on Proportional Integral (PI) controllers with feed-forward cross-coupling terms applied to the d-q coordinates in the current and voltage control loops. It is worth noting that the swing equation (and not the PLL) provides the frequency and phase angle references to the control system, thereby implementing the virtual inertia behaviour. The active power control is provided by an outer steady-state frequency droop controller, as shown in Figure 3. A reactive power droop contributes to the voltage magnitude reference as depicted in Figure 4.

Note that ω_{ref} , ω_{PLL} , and ω_{VSG} , are the reference speed, measured speed from the PLL, and rotating speed (angular frequency) of VSG. The angular position of the VSG, θ_{VSG} , corresponding

to the phase angle of the converter voltage reference, is given by the integral of the angular frequency. M and D are the inertia constant and the damping constant, respectively. The P_d is used to provide active power damping in case of frequency deviation.

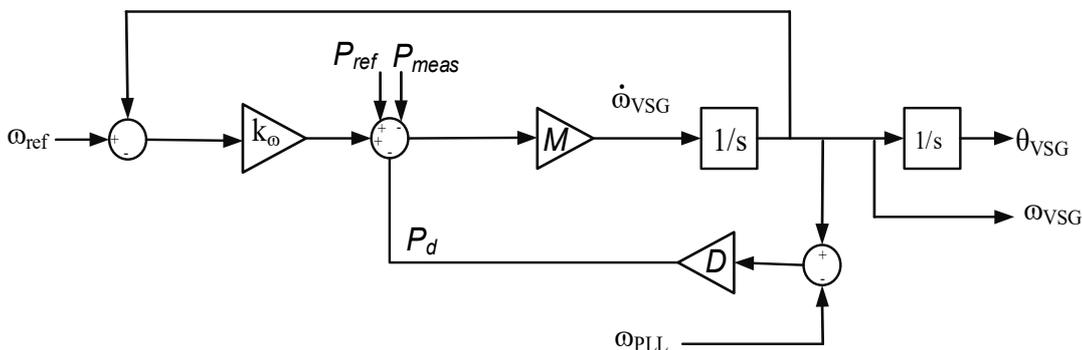


Figure 3. Inertia emulation control loop (active power-frequency droop control)

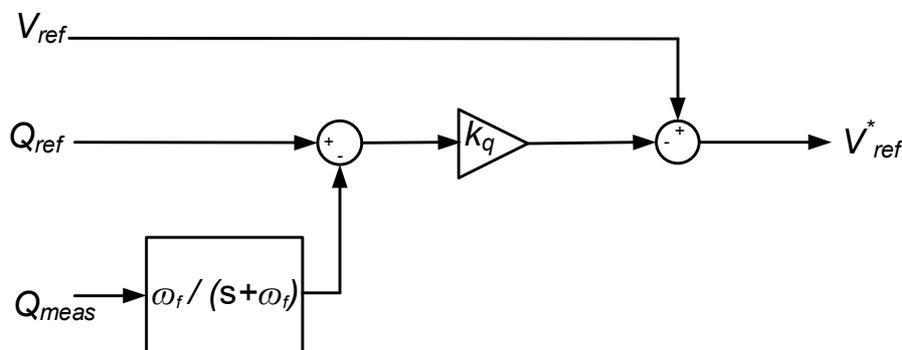


Figure 4. Reactive power-voltage droop control loop

In Figure 3, $1/s$ is the differentiator used in the control scheme, ω_{PLL} is the measured speed from the PLL, ω_{VSG} and θ_{VSG} are the reference speed and angle provided by the VSG. The gain K_q is the reactive power droop gain acting on the difference between the reference and measured reactive power. The term $\omega_f / (s + \omega_f)$ is a first order low pass filter, where ω_f is the cut-off frequency.

The equations, control structure, and detailed explanation of PLL and additional active damping terms are explained in detail in [0] and [13].

3.2 Synchronverter

The SV is a converter with a specific embedded control algorithm that mimics the operation of a synchronous generator [9] and [10]. The model of a SV is derived from the model of a three-phase round-rotor SG as it behaves in the same way, mathematically, to provide the voltage supply. The SV has the capability of frequency and voltage regulation through the control of active and reactive power, respectively [6]. The SV achieves the synchronization with the power grid without a dedicated synchronization unit which is an identical feature of SG. The SV is composed of two parts: power and electronic (control) parts, similar to VSG. More details about SV structure and control are provided in [9] and [10].

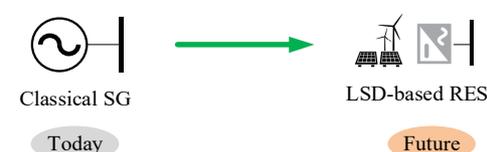
4. Towards Linear Swing Dynamical Systems

4.1 Overview

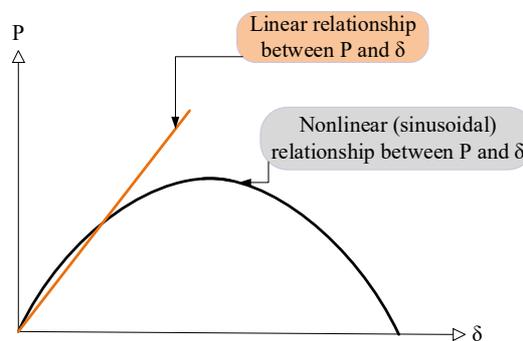
In order to contribute to the establishment of future converter-based power systems requirements, the concept of Linear Swing Dynamics (LSD) is proposed as depicted in Figure 5. The aim is to: provide a linear dynamical system with highly-predictable and controlled dynamics, enhanced system stability and dynamic performance. Also, this concept facilitates analysing system stability with capturing the stability information of the whole state space, i.e. small and large disturbances.

The LSD concept consists in linearizing the nonlinear (sinusoidal) power-angle ($P-\delta$) characteristic, which in turn, describes system swing dynamics during power disturbances. The respective LSD control adopts the new (linear) power-angle characteristics and formulation, to be implemented and embedded in the control of RES-tied converters.

The up-to-date control schemes proposed for power converters are based on generic droop control and/or SG emulation (VSG and SV). The droop control does not provide the inertial response, nor generation-system synchronization and coherency. Also, a stable frequency profile at one stable operating point cannot be achieved/guaranteed in systems with 100% converter-based generation. On the other hand, the VSG/SV adopts the classical representation of a SG, and hence, does not provide LSD achievements.



(a)



(b)

Figure 5. a) future scenario. b) Power-angle characteristics, considering the proposed linear swing dynamics.

The LSD will be embedded into the VSG as illustrated in Figure 6. [15]. The aim is to take the advantages of the SG characteristics (system synchronization, droop control, and inertial response) and tackle its downside represented by nonlinear characteristics (nonlinear swing dynamics). It is worth mentioning that the proposed LSD-VSG aims to maintain the same behaviour of today's power systems in a future with 100% converter-based generation. Hence, to guarantee that all the deployed converter-interfaced RESs are synchronized under different operating conditions. This in turn, preserves system stability, coherence, and dynamic performance. Note that the difference between the LSD-VSG shown in Figure 6 and the VSG in Figure 4 is that the former does not use reactive power loop to regulate the voltage. Instead, LSD loop and Virtual Impedance (VI) loops are proposed

The proposed LSD-VSG enables a grid-friendly converter for RES integration in future power systems, and has the following objectives:

- Achieve linear and uniform system swing dynamics. This is due to the linear characteristics of LSD, in contrast to the nonlinear characteristics of conventional SG.
- Provide virtual inertia and participate with system frequency regulation.
- Regulate the voltage through an outer voltage control loop.
- Maintain generation-system synchronization and coherency, by enabling the LSD-VSG to act actively, as in 2-3, in case of disturbances.

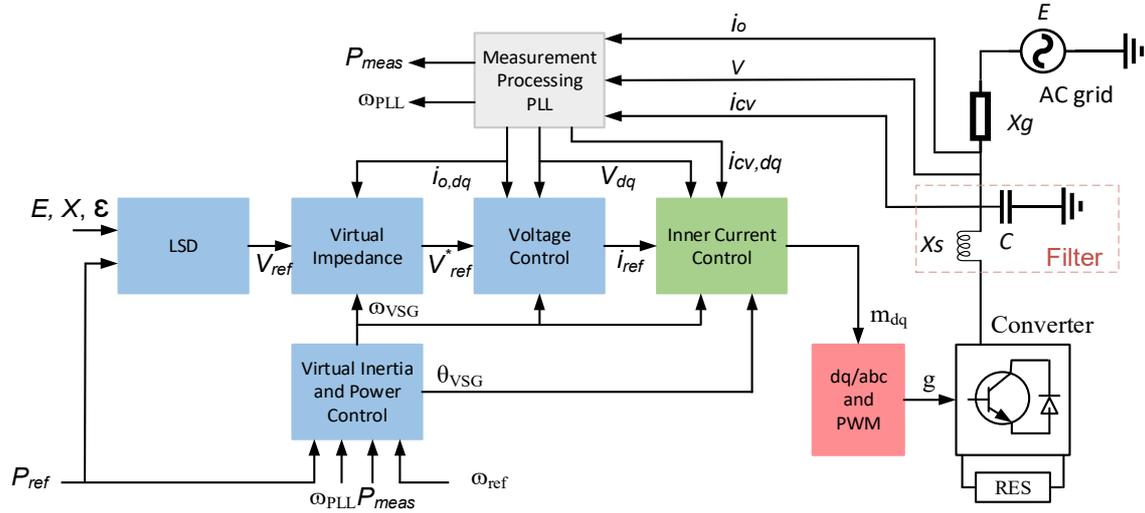


Figure 6. Schematic diagram of LSD-VSG

4.2 Linear swing dynamic definition

In order to derive the LSD concept, let us consider the system shown in Figure 7:

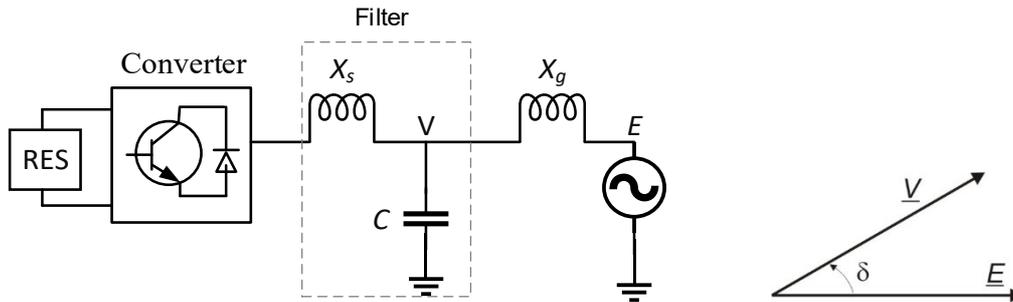


Figure 7. Inspected Single Machine Infinite Bus (SMIB) system.

The studied system consists of a three-phase Pulse Width Modulation (PWM)-controlled Voltage Source Converter (VSC), interfaced to a Thevenin-equivalent system by an LC filter.

By linearizing the solution of swing equation and power angle formulas, which are provided in the Annex, around an operating point, the following is obtained:

$$\begin{bmatrix} \Delta \dot{\delta} \\ \Delta \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{EV}{XM} \cos \delta & -\frac{D}{M} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \end{bmatrix} \quad (1)$$

where ω is the rotational speed, and δ is the angle between E and V .

The dynamic behavior of this system depends on the actual operating point (P), and has a nonlinear dependence on δ .

The goal is to have a linear relationship between P and δ . Since E and X are constant, V can be regulated by appropriate control to facilitate achieving LSD characteristics, i.e. linear relationship between P and δ . Note that X here includes both the generation/converter side reactance (X_s) and the system reactance (X_g).

This can only be done by an appropriate control of V , considering that V has to be maintained within $V_{\text{nominal}} \pm \text{tolerance } \epsilon$ (5% for Transmission System (TS) with voltage levels above 220 kV, and 10% for Distribution System (DS) and high-voltage systems up to 220 kV) [14]. It can be observed that the role of ϵ is to provide a degree of flexibility in voltage variation to achieve the LSD concept, i.e. linearized power-angle characteristics.

The proposed solution is based on voltage control, along with virtual impedance loop. The control law can be derived as follow

$$P = \frac{EV}{X} \sin \delta = \frac{E}{X} (1 - \epsilon) E \delta \quad (2)$$

From eq. (2), it is possible to calculate δ and V as follow:

$$\delta(P) = \frac{PX}{(1 - \epsilon)E^2} \quad (3)$$

$$V(\delta) = \frac{(1 - \epsilon)E}{\sin \delta} \delta \quad (4)$$

where the rotor angle (for the corresponding reference active power) and the variable reference voltage are given by (3) and (4), respectively.

It can be shown that for $\epsilon=0.1$ and by using (3) and (4) the $P(\delta)$ relationship will be linear in the angle range $[0^\circ, 62^\circ]$, the corresponding power range $[0, 0.97]$ and voltage range $[0.9, 1.1]$, as shown in Figure 8.

For $\epsilon=0.05$ and by using (9) and (10), the $P(\delta)$ relationship will be linear in the angle range $[0^\circ, 44^\circ]$, the corresponding power range $[0, 0.73]$ and voltage range $[0.95, 1.05]$. This is illustrated in Figure 9.

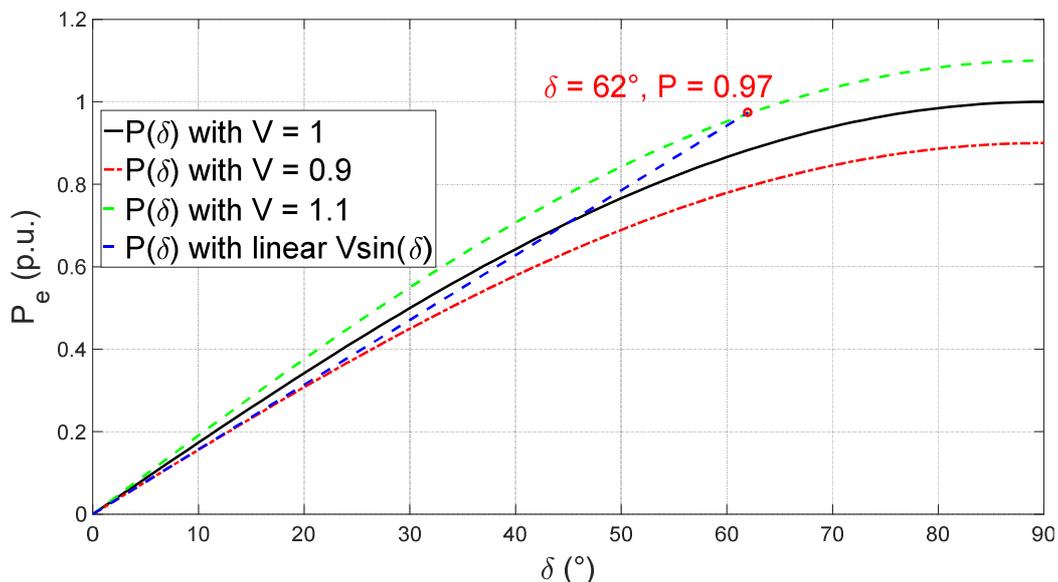


Figure 8. Proof of concept with $\epsilon = 0.1$.

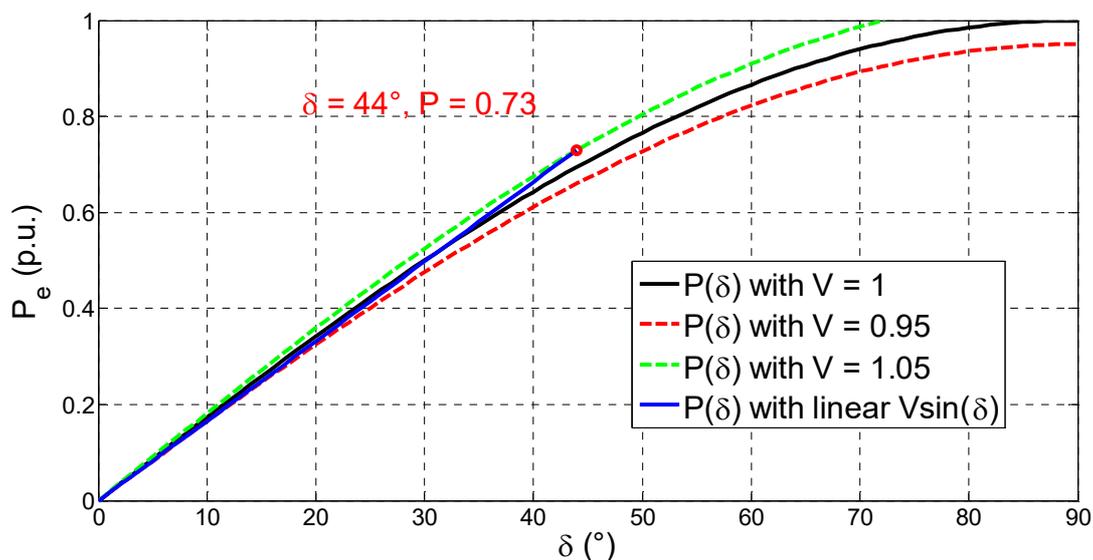


Figure 9. Proof of concept with $\varepsilon = 0.05$.

In some cases, the power transformers in the system would limit power flows, so that these would not exceed the maxima 0.97 (0.73) indicated in Figure 8 (Figure 9).

However, we are currently analysing the situations when the power-angle values (temporarily) exceed the indicated maximum values, and we are also investigating the possible methods to deal with such situations. (Possible methods could include: using piece-wise linear approximation, temporarily allowing voltage limit violations, or applying LSD techniques that do not rely on voltage control.)

It is worth mentioning that from theory point of view, the LSD characteristics can be achieved till infinity, i.e. beyond $\delta = 45^\circ$ (65°) in TS (DS). However, the LSD concept relies on the voltage control and the permitted tolerance, ε , which cannot be exceeded it in practice.

The LSD control law can be achieved by either:

- setting V_{ref} according to eq. (1) and eq. (4) and setting the VI values equal to zero: in this case $V_{ref} = V_{ref}^*$ (as shown in Figure 6).
- or keeping V_{ref} constant, and designing the VI control so that the voltage reference input of the voltage control loop (V_{ref}^* in Figure 6) be always the same as in eq. (4).
- We are currently investigating other methods, which also do not require the change of V_{ref} .

5. LSD-based Improved Virtual Synchronous Generator

5.1 Control configuration

The solution of LSD-based VSG (LSD-VSG) is proposed for the RES-tied converters. Unlike a traditional VSG, the reference voltage command is not introduced to be a constant as in Figure.6. Instead, it is being calculated in the newly introduced LSD loop. The control scheme of LSD-VSG employs d-q coordinates transformation in order to ensure decoupling characteristic through virtual impedance and other feed forwards loops. The VSG loop emulates the steady state control characteristics of SG, maintains the converter's output frequency by comparing the reference and generated active power governed by the $P-\omega$ equation (active power-frequency droop control), and emulates virtual inertia. The LSD loop provides the updated voltage reference, based on LSD concept expressed in (9) and (10), to be fed to the VI then voltage outer control loop.

As depicted in Figure 6, the outer voltage control and inner current control loops are constructed according to the generic control structure of VSC, so that the d-q components of voltage and current can be controlled separately through PI-controllers. Moreover, the filter LC-oscillations at the grid side interface are actively damped. The LSD-VSG produces d-q axis components of the output voltage which is then transformed to three phase system and subsequently fed into the converter's modulation stage. As shown in Figure 6, X_s and C are the converter (LSD-VSG) LC filter, and X_g is the AC grid impedance.

The presented LSD concept does not contain any explicit means for reactive power control. (i.e. the reactive power control loop in [19] is bypassed.) However, from the practical point of view, we do not consider this a problem, since reactive power control is used in practice mostly for voltage control purpose. The presented LSD concept relies on explicit voltage control, and therefore already contains the means to keep system voltages between the prescribed limits. This is the reason why the reactive power control loop is not included in the LSD-VSG. In contrast with the classical VSG shown in Figure 4.

It is worth mentioning that the VI loop is implemented simply by calculating the virtual impedance ones based on the corresponding AC grid impedance. Hence, there is no consideration of (closed-loop) VI control. More information about the virtual impedance control can be found in [16].

5.2 Simulation scenarios

For concept validation, the proposed LSD-VSG has been tested, under different operating conditions, in the SMIB system shown in Figure 7. The model parameters of SMIB system are listed in Table 1.

During the simulation, the active power reference has been changed to emulate power disturbance scenarios, which are caused by an increase in power demand. The active power is increased from 0.01 to 0.02 pu in the time instant 4 s, and further changed with step 0.2 pu in successive time instants (i.e. 8, 12 and 16 s) to reach 0.8 pu, which corresponds to 400 KW. This is shown in Figure 10.

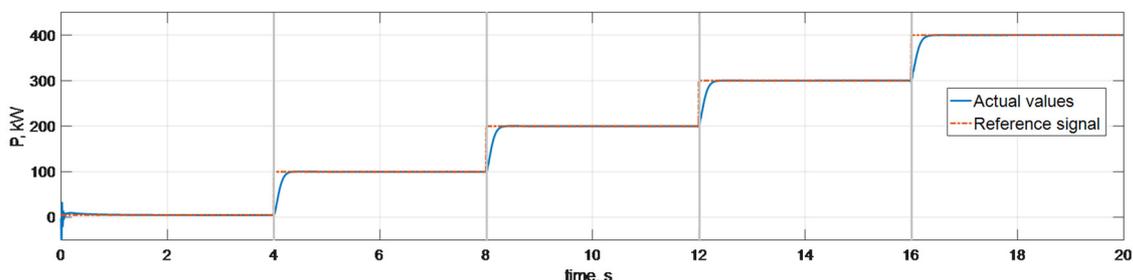
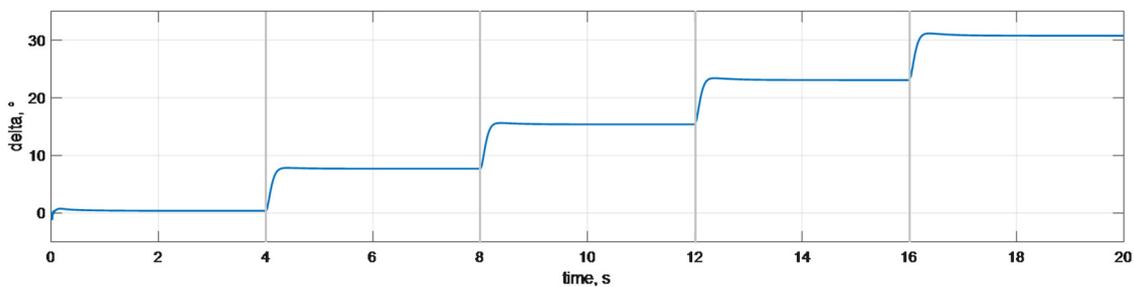
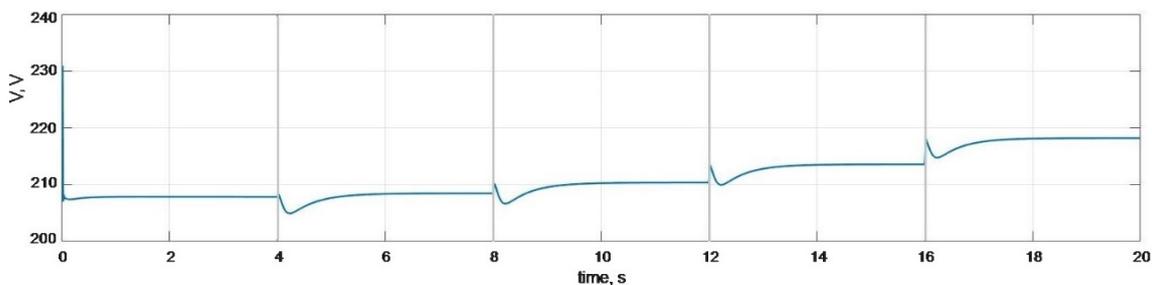


Figure 10. Active power delivered by LSD-VSG.

Table 1. Model parameters for SMIB simulation

Parameter	Value
Rated Voltage (E)	400 (v)
Rated Power (S)	500 (kVA)
Rated angular freq (ω).	$2\pi 50$
VSG Inertia constant (M)	0.0005
VSG damping coefficient (D)	40000
Filter inductance (L_s)	10 (mH)
Filter resistance (r_s)	0.163 (Ω)
Grid impedance (L_g)	615.4 (μH)
Grid resistance (r_g)	0
Droop gain (k_ω)	$S/(2\pi)$
DC side voltage (V_{dc})	5700 (v)
Voltage tolerance (ϵ)	0.1 pu.

**Figure 11. Phase angle (δ).****Figure 12. Output voltage of LSD-VSG**

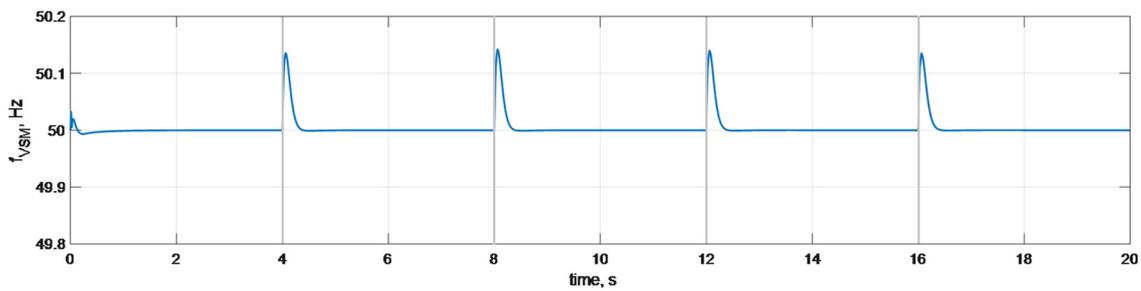


Figure 13. Frequency of LSD-VSG.

The considered power disturbances caused a deviation in δ with up to 33° as depicted in Figure 11. The steady-state $P(\delta)$ value pairs are found at the linearized curve, according to the LSD control, which in turn, adjusts the voltage value with acceptably and small transients depicted in Figure 12.

During power disturbances, the LSD-VSG was able to regulate and stabilize the frequency, as observed in Figure 13. However, in this study, unlimited power reserve (source) is considered from the DC side of the LSD-VSG, as the focus is on the development of LSD concept and its control. Later, this source will be changed with more realistic representation of storage-connected RES.

From the obtained results shown in Figure 14, it can be observed that for an increase in active power generation, a linear transition is observed in the LSD characteristics, as depicted in the blue line. This is in contrast with classical SG (or VSG without LSD), in which a sinusoidal (nonlinear) characteristics is observed in the green line.

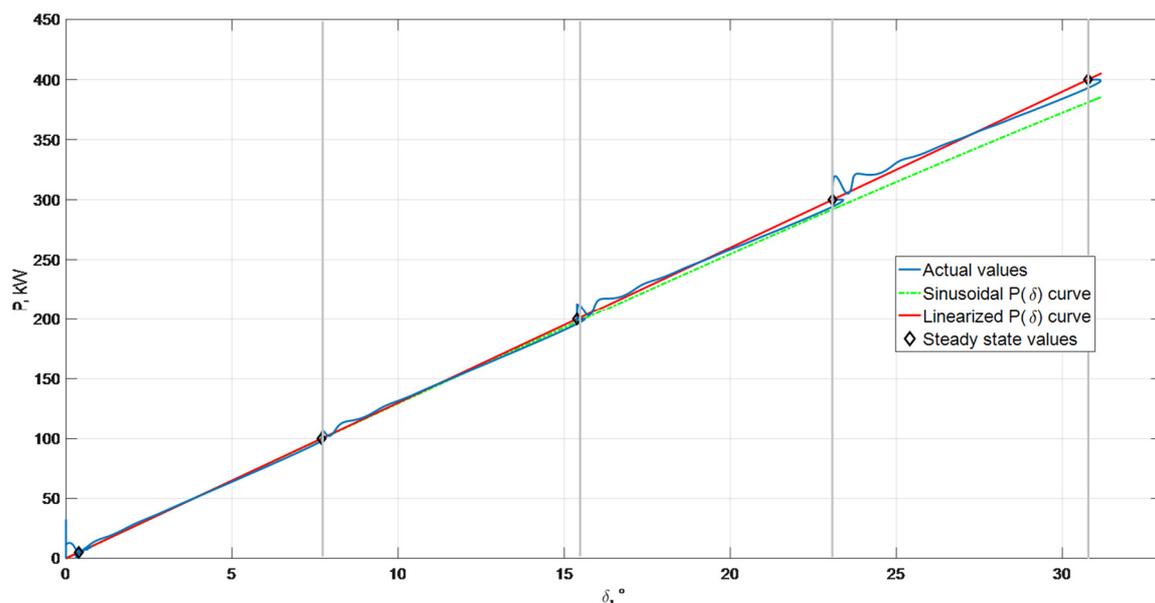


Figure 14. Power-angle characteristics.

5.3 Stability analysis of LSD-VSG system

The small signal stability analysis is conducted to assess system's behaviour and investigate its dynamic characteristics and operation modes, with the consideration of various control parameters. This is done by linearizing the system around the Steady State (SS) operating point by calculating a Jacobian matrix from the canonical representation of the system, for a specified set of inputs, and calculating system's eigenvalues. As already observed from the time-domain simulations, the system is stable with the specified control parameters, under different operating conditions [17-19].

The small signal model has been developed for the LSD-VSG system, based on the respective mathematical formulation. The state-space representation and linearization of the LSD-VSG

includes: power and electrical circuit, Phase Locked Loop (PLL), virtual inertia (VI), outer voltage control, and inner current control loops. Note that the LSD-VSG control structure is developed on the bases of Synchronous Reference Frame (SFR) as shown in Figure 21 in the Annex A.2. Also, the difference between the SFRs orientation angles is used to define the phase angle between PLL and VSG angles. Furthermore, all the control loops employ a reference frame based upon the angular position dedicated by the VSG loop. Hence, in order to develop a mathematical model of the system, the mathematical equations must be developed in similar reference frame oriented upon VSG [20].

The LSD-VSG system has total of nineteen eigenvalues (poles). With the initially set control parameters, the system has: poles reside in close vicinity of the origin, complex conjugate pole pairs with relatively poor damping and high oscillation frequency, and another complex conjugate pole pair with a relatively good damping and low oscillation frequency. This is shown in Figure 15. Then, they are initially tuned according to two of the conventional tuning techniques, named modulus optimum and symmetric optimum [21-23]. Afterwards, they are further refined using the conclusions taken out from the eigenvalue analysis shown in Figures 15-18, and time-domain simulations. The presented stability analysis is performed by studying the influence of control gains (parameters) selection on system stability. Referring to Figure 6, the following control parameters are studied: k_{pi} and k_{pu} which are related to the PI of voltage control, and k_{ij} and k_{iu} which are related to the PI of inner current control.

To analyse the trajectory of the eigenvalues, the following is observed:

- It is observed that with decreasing the proportional gain, k_{pi} , of voltage PI control, the conjugate pole pairs move far away from the real-axis, which translates in decreasing the speed of the control system. The critical poles near the origin can only be placed away from the origin to a certain extent, as depicted in Figure 17.
- The increase in voltage control gain, k_{pu} , affects mainly the poles which are located far from the origin. The significant increase in k_{pu} assembles these poles towards a single point, and decrease their oscillatory frequency. This is illustrated in Figure 16.
- When the integral gain of the inner current controller is swept, k_{ij} and k_{iu} , it pulls the group of complex conjugate pair toward the stable region. At the same time, it pushes the real eigenvalues towards the origin. This is shown in Figure 18. Hence, the selection of proper values, for k_{iu} and k_{ij} , is a tradeoff, and the selection should be adjusted to have satisfactory damping from the complex conjugate pairs poles. At the same time, keep the real eigenvalues away from the origin (zero).

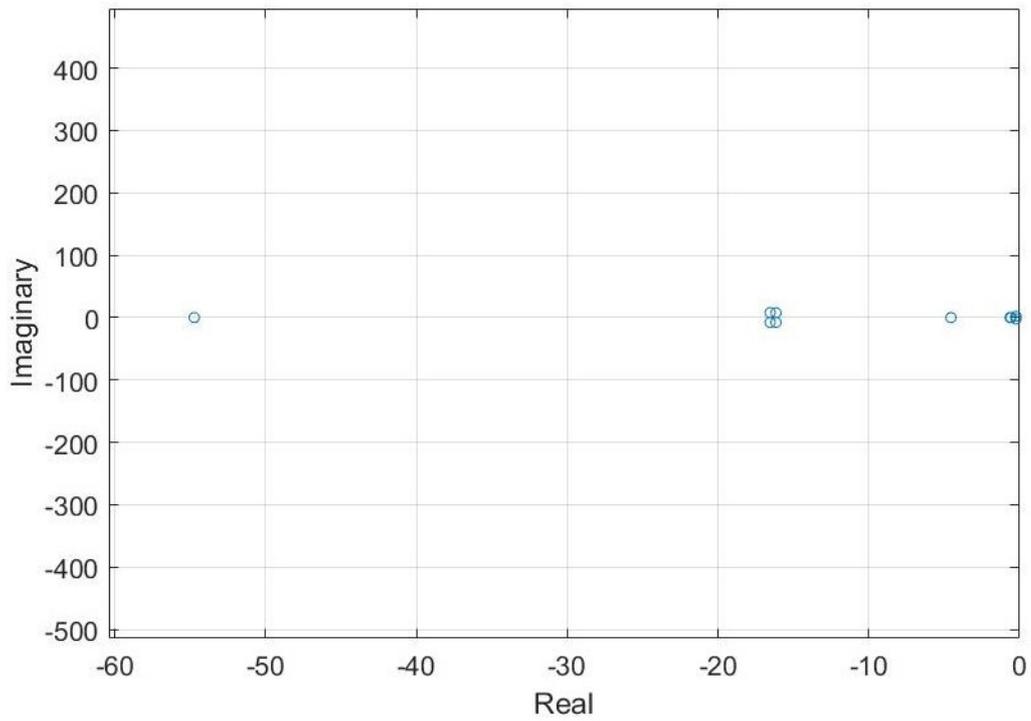


Figure 15. System eigenvalues.

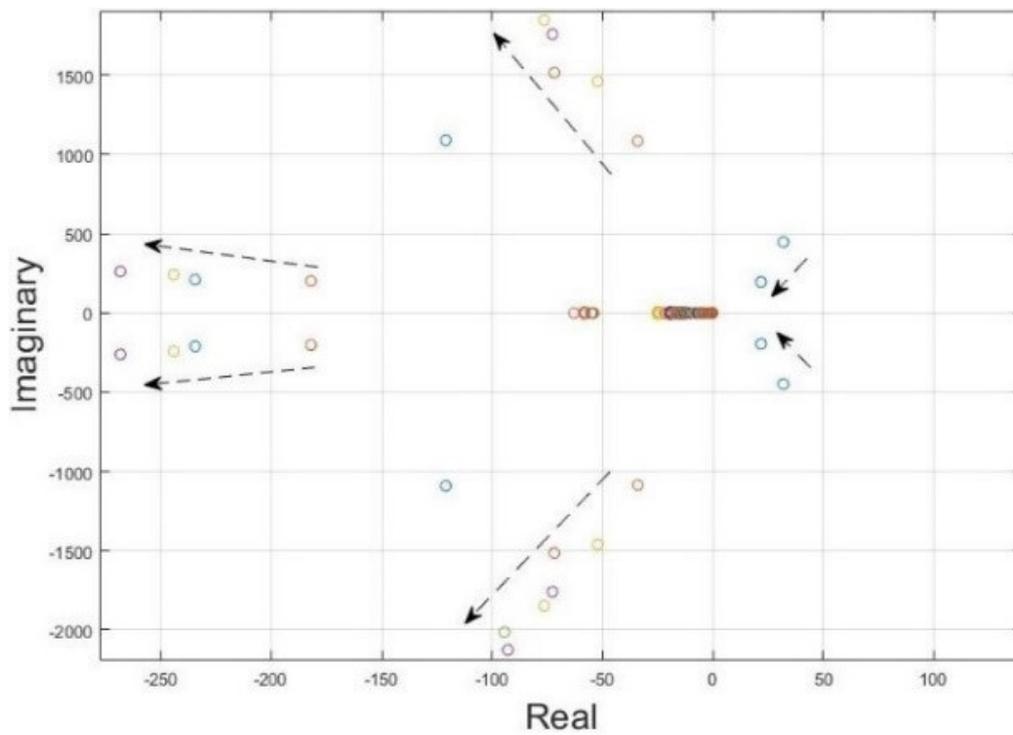


Figure 16 - Root locus of K_{pi} parameter of voltage control loop

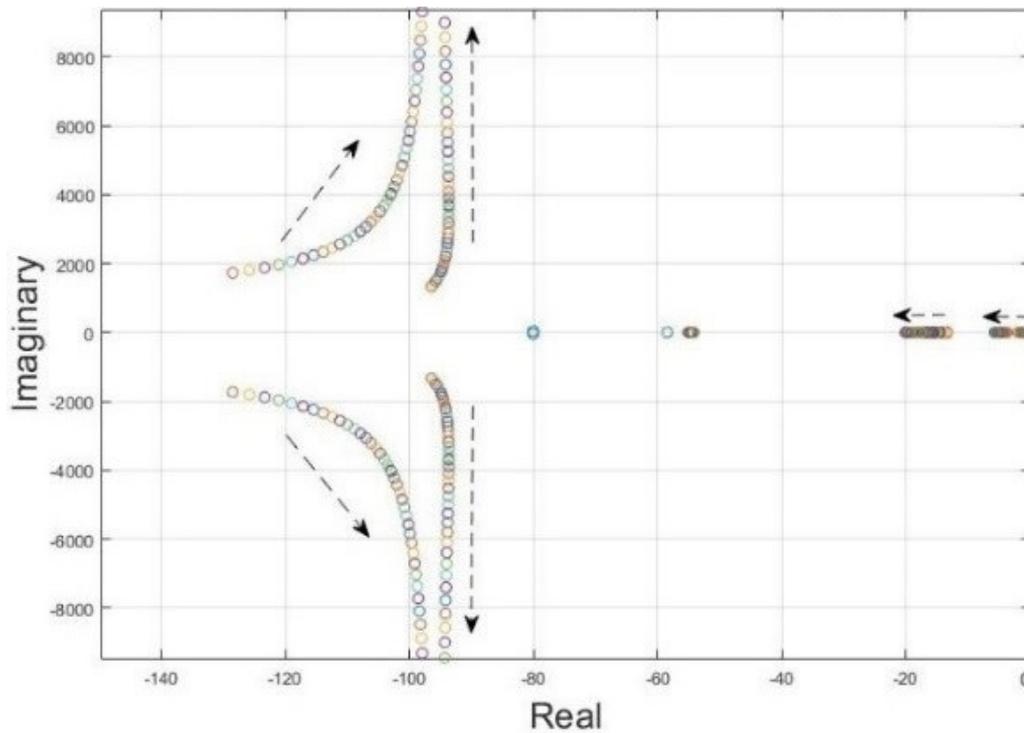


Figure 17. Root locus of Kpu parameter of voltage control loop

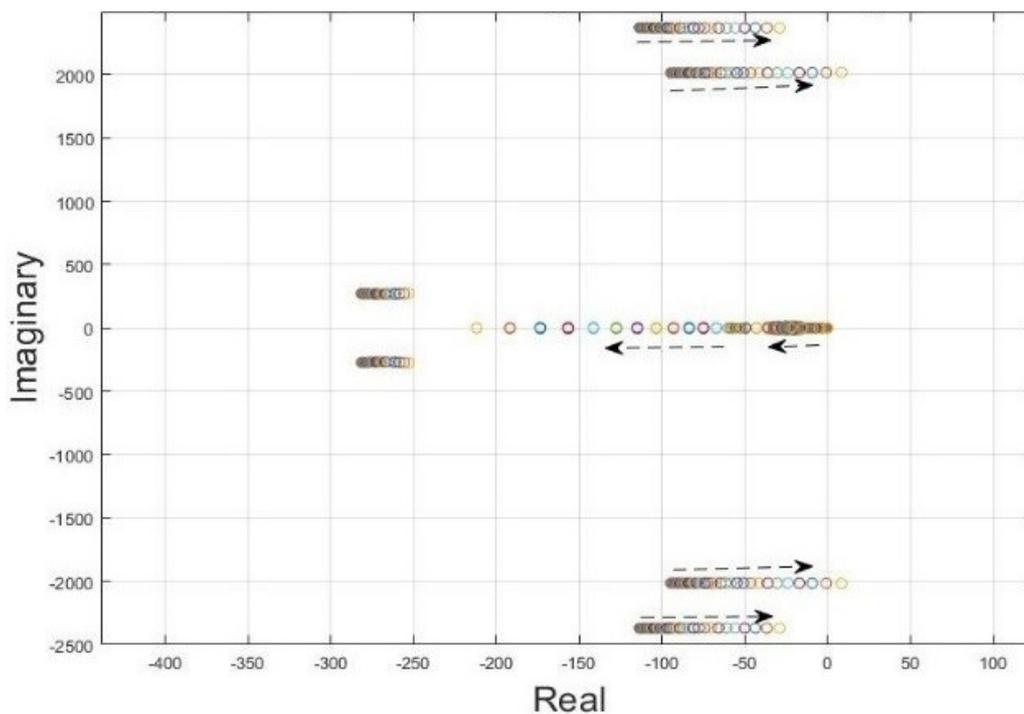


Figure 18. Root locus of Kiu and Kii parameters of inner current control loop.

5.4 Implication of LSD on system communication and automation architecture

The very high deployment of converters in future power systems, on both generation and load side, will result in:

- Very fast dynamics;

- Reduced time frame for system operation and control, comparing with today's power systems. This is due to the fast response of the converter-interfaced RES, in contrast with the SG which has slower response due to the respective mechanical motions;
- System reliability and performance will rely thoroughly on the converters' control;
- Necessity for a fast, robust, and reliable communication architecture due to eqs. (1-3) .

The communication architecture will be integrated to coordinate the control activities. This includes a bi-directional transmission of measurements and control signals among the local LSD-VSG (primary controllers), and from primary to supervisory control (secondary control). However, the selection of communications technology and topology depends on the geographical area of the power grid, e.g. transmission or distribution network, and the respective control architecture, e.g. decentralized or coordinated control. Figure 19 illustrates the control and communications architecture in a 9-bus system. The control architecture is composed of coordinated Primary Control (PC) and Secondary Control (SC) levels. The PC is getting the measurements of active power (voltage and current), virtual angle, frequency (speed) and deviation in frequency (Rate of Change of Frequency - RoCoF) from the respective Point of Common Coupling (PCC) and Phase Locked Loop (PLL). Also, it receives specific information/measurements, via communications lines, from other PCs.

It is worth mentioning that an inertial control is implemented in a decentralized way, in each LSD-VSG, to provide virtual inertia in case of disturbances. In this regard, an additional storage unit is connected on the DC side of RES. The storage can participate in discharging/charging its energy in case of under frequency/ over frequency scenarios, respectively. More details about the storage control and participation in frequency support are provided in [24]. The depicted system is currently under study, and the results will be reported in the near future.

The following information are required as inputs to the communications system:

- Number of converters (LSD-VSG), measurement units (PMU, PLL or frequency divider), and control centers;
- The PC and SC control architecture;
- Distances among the controllers and measurement units;
- Data time flow;
- Latency and sampling rate.

Based on the above information, the proper communications technology will be chosen, e.g. fibre optic or wireless (5G) technology. Discussion on the requirements placed on communications systems for future energy systems can be found in [25].

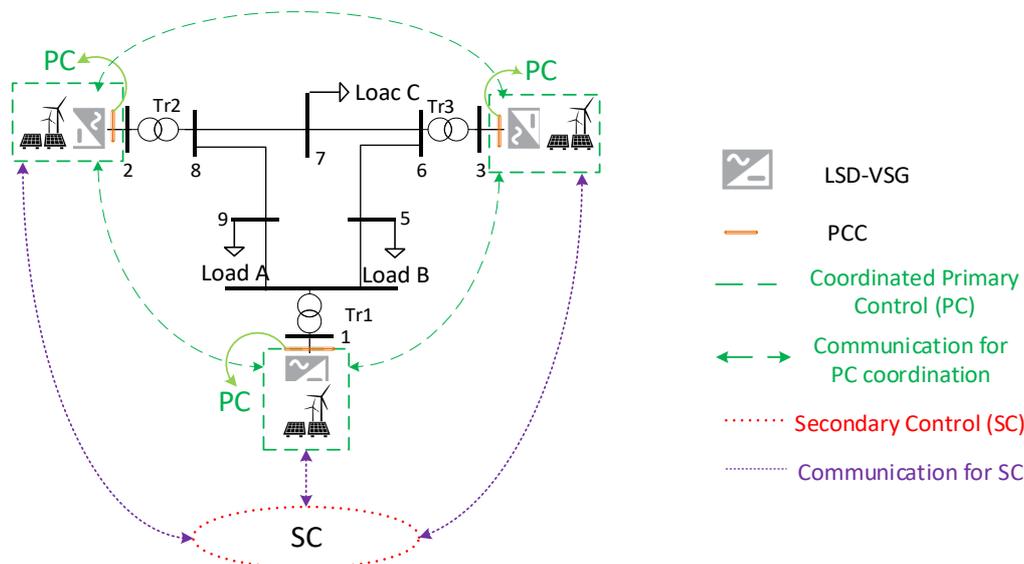


Figure 19. Control and communication in a 9-bus system.

5.5 Implication of LSD on network codes

The LSD concept is proposed for future power systems with up to 100% generation from RES, and the respective LSD-based control (LSD-VSG) is developed to be implemented in the RES-tied converters. It is worth mentioning that due to the increasing interests in HVDC systems considering their technical features for system frequency support, a new control scheme based on LSD concept is proposed for the HVDC converters as well. The aim is to embed the LSD concept in all the power source converters, which are related to RES and single/multi-infeed HVDC systems to achieve: linear dynamical system operation, overall AC/DC system synchronisation and coherency, and consistent frequency control performance. Based on this scenario, preliminary recommendations are obtained for the network codes as follow:

- New requirements and roles for the RES-tied converters' control and frequency regulation. This includes system operation with a voltage tolerance greater than 5% (10%) in TS (DS). Also, the necessity of connecting an Electrical Storage System (ESS) with the RES-tied converter to enable the latter participating in virtual inertia provision.
- New requirements for HVDC systems that respect the technical constraints and specifications of each AC network. Furthermore, HVDC systems should provide synthetic inertia to the disturbed AC network without compromising the frequency stability of other HVDC-connected AC networks. These activities will definitely consider the coordination between HVDC system owners and the respective TSO.
- In future power systems, the inertial response of the synchronous machines will progressively play a residual role in the regulation of the frequency in the first instants after a contingency. As a consequence, RES, ESSs, HVDC will be expected to provide ROCOF control thanks to their fast responses, in addition to primary frequency control. To this aim, it is imperative to establish a practical definition of ROCOF such that it can be used as an input signal for the regulators of power converter-based devices.

6. Conclusion

The LSD concept is proposed for future converter-based power systems, and its respective control has been developed for RES-tied converters (LSD-VSG). The aim is to exploit the features of fully-controlled converters to preserve the advantages of classical SG and tackle its disadvantages. The objectives of the developed LSD-VSG are: achieving linear dynamical system with linearized power-angle characteristics, maintaining generation-system synchronization and coherency, and participating in frequency support. Also, the proposed LSD-VSG has the inherent capability of regulating the voltage, under different operating conditions, to be maintained within the acceptable tolerance. The LSD-VSG has been tested and validated in SMIB system. The results fulfil the above-mentioned advantages and confirm the significance and substantial role of LSD (LSD-VSG) in future converter-based power systems.

Stability analysis is performed to investigate the dynamics and operation modes of the LSD-VSG system. Then, the respective control parameters are tuned, based on the conducted stability analysis and time-domain simulations, to attain better damped response with an enhanced dynamic performance.

Preliminary recommendations and specifications on communications requirements and network codes are discussed based on the conducted research work and the obtained results.

Future work

This deliverable focused on the development and validation of LSD concept and the proposed LSD-VSG in a simplified test system. However, the concept will be further studied and verified considering the following future work:

The E and X values are calculated simply by considering the Thevenin equivalent system. However, they can be calculated in a more sophisticated approach for larger (interconnected) power systems, considering faults and change in system topology. In this regard, the dynamic equivalent impedance of a multi-machine system is being studied.

- The proposed LSD-VSG is tested in a SMIB system. In future work, the LSD-VSG will be improved and tested in a multi-machine power system, e.g. WSCC 9-bus system and the Romanian Transmission network, considering the outcome of (1).
- The developed LSD-VSG is a decentralized control with a pure local measurements, i.e. there is no coordination and communication considered. In future, coordinated LSD-VSG control will be investigated, along with the specifications and requirements of communications architecture.
- The LSD concept is proposed for transmission system, with the consideration of $Z \approx X$, $X \gg R$. In future work, the LSD concept will be developed for distribution systems where $Z = R + X$.
- As the HVDC systems will have significant roles in future power systems, the LSD-based HVDC system will be studied. The aim is to provide ancillary services from LSD point of view, e.g. frequency support and power management, to the adjacent AC power systems with respecting their technical specifications and constraints.
- The LSD-based Synchronverter will be developed, and the same analysis and process will be repeated to LSD-SV as in LSD-VSG.

The results and conclusions of the above research work will be reported, as part of Task2.3, in future deliverables within WP2.

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8. List of Abbreviations

DS	Distribution System
LSD	Linear Swing Dynamics
MMF	Magneto-Motive Force
PCC	Point of Common Coupling
PC	Primary Control
PLL	Phase Lock Loop
PMU	Phasor Measurement Unit
PWM	Pulse Width Modulation
RES	Renewable Energy Systems
RoCoF	Rate of Change of Frequency
SC	Secondary Control
SG	Synchronous Generator
SMIB	Single Machine Infinite Bus
TS	Transmission System
SV	Synchronverter
VI	Virtual Impedance
VSC	Voltage Source Converter
VSG	Virtual Synchronous Generator
WSCC	Western Systems Coordinating Council

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10. Annex

A.1 Power system stability

Power system stability can be classified as shown in Figure 20

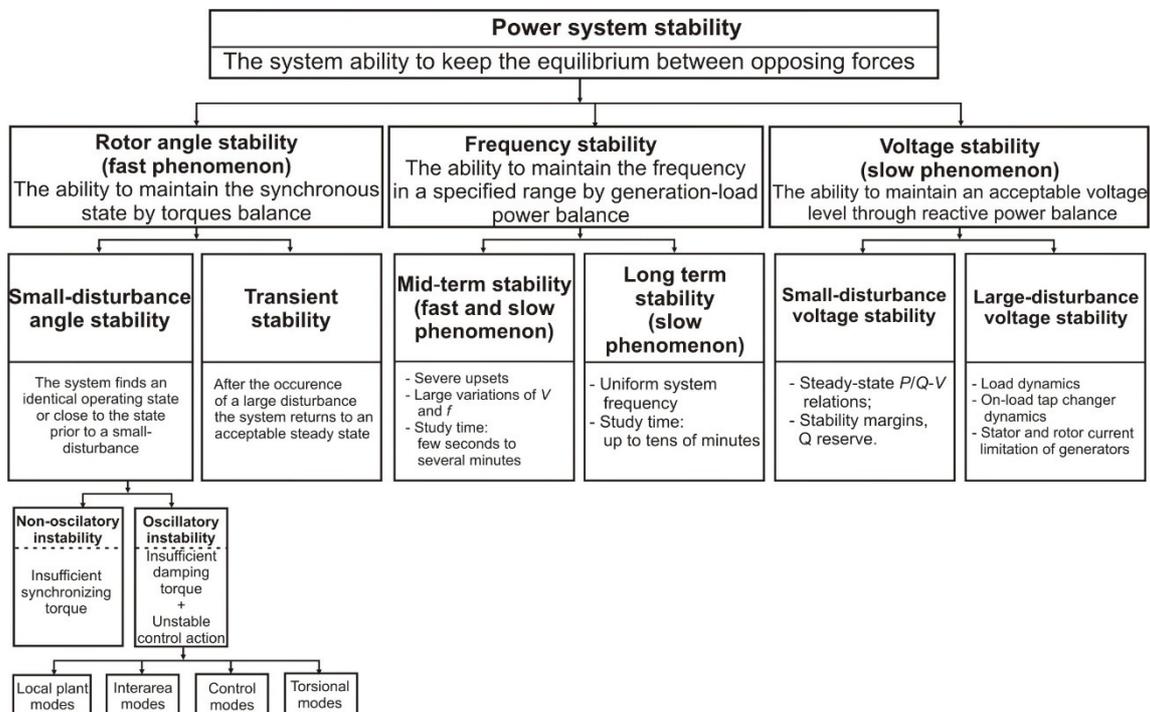


Figure 20. Classification of power system stability.

A.1.1 Small signal stability

The small signal stability is related to the ability of the power system to maintain the synchronism under continuous, but normal, small disturbances originating from both the generation and load sides. The small disturbance could be the action of an automatic voltage regulator (AVR) or normal power fluctuations in load. System linearization can be used to analyse its stability around a specific (limited) operating margin.

A.1.2 Large disturbance stability

Large-disturbance angle stability, commonly referred to as transient stability, is concerned with the ability of a power system to maintain the synchronism, following a severe disturbance, such as a transient fault, disconnection of a large power plant or disconnection of a large load. In such transient conditions, the SG power (load) angle changes rapidly due to the sudden acceleration of the rotor shaft. In this regard, the transient stability analysis is conducted to ascertain whether the load angle returns to a steady value after disturbance clearance.

The above conditions increase the complexity of analysing system transient stability, in which the linearized system is not applicable, as it can be done for small signal stability. For this reason, iterative analysis is required for different disturbances to get the proper transient stability profile (information) of the system. Note that maintaining system stability is more important under high loading conditions of the machines, as the load increase can bring the onset of instability [4].

A.1.3 Frequency stability

Frequency stability is concerned with the ability of the power system to maintain the frequency within a pre-set range following a severe disturbance consisting in sudden and large unbalances between generation and load. Therefore, it is the ability of the power system, eventually with the

intervention of the human operator, to restore the balance between generation and load with minimum loss or load. Since the generator can automatically react to frequency changes, the critical powers unbalance situation is that when the generated powers are smaller than the total load [26].

Instability results in an uncontrolled sustained increase or decrease over time (a “run-away” condition) or sustained undamped oscillatory behaviour [27]. Since the frequency is mainly a global power, the frequency stability depends directly on the size of the power system in terms of the instantaneous generated/consumed powers.

Analysis of frequency stability is performed by time-domain simulations, and the disturbances should be set in such a way to create both fast and slow dynamics for which specific protection and automation systems are designed. This type of the disturbances is decided in terms of the size of the power system.

As the synchronous generators are replaced by power electronic converter interfaced power sources, the frequency dynamics are changing because of the reduced mechanical inertial. As shown in deliverables D2.1 and D2.2, there is a need for new technologies and new control techniques, designed to react in the case of the fast dynamics.

A.2 Rotational inertia and swing equation of synchronous generator

Due to the electro-mechanical coupling SG, the respective rotating mass provides kinetic energy, which in turn translates to a rotational inertia, to the grid (or absorbs it from the grid) in case of under (over) frequency problem caused by power imbalance (disturbance) scenario. The contribution of inertia is an inherent and crucial feature of SG, and is an important property of frequency dynamics and stability.

The motion of the SG rotor (rotating mass) is described by the following equation

$$J \frac{d^2\theta}{dt^2} = T_m - T_e = T_a \quad (5)$$

Where J , θ , T_m , T_e , and T_a are the moment of inertia, angular position of the rotor with respect to a stationary axis, mechanical torque, electrical torque, and the accelerating/decelerating torque.

By multiplying both sides of (5) by the nominal rotor speed, ω , the following is attained

$$M \frac{d^2\theta}{dt^2} = P_m - P_e \quad (6)$$

where $M = J \omega$ is the angular momentum. P_m and P_e are the input mechanical power and output (generated) active power of a SG. Note that θ can be expressed as

$$\theta = \omega t + \delta_m \quad (7)$$

where δ_m is the rotor angle with respect to a synchronously rotating reference frame with velocity ω . Substituting (6) in (7), the following is obtained

$$M \frac{d^2\delta_m}{dt^2} = P_m - P_e \quad (8)$$

From the equations (5), (6), it can be observed that the original form of nonlinear swing equation increases the complexity in solving/ analysing system stability as well as the development of system frequency control.

After some formulations, the following equation is obtained which represents the swing equation in per unit

$$M \frac{d^2\delta}{dt^2} = P_m - P_e \quad (9)$$

where δ is the load angle = $\delta_m \frac{p}{2}$, p here is the number of poles for the SG

Eventually, the expression for the active power ($P = P_e$) is given us

$$P = \frac{EV}{X_g + X_e} \sin \delta \quad (10)$$

where V and E are the generator and grid side voltages, respectively. The X_g quantity is the generator reactance (transient reactance used for dynamic studies), and X_e is the power system reactance (considering the SMIB system) [2]-[4]. It is worth to be noted that the swing equation (8) with P_e expressed in (10) is a nonlinear differential equation for which there is no analytical solution in general. Note that (10) shows the sinusoidal relationship between P_e and δ , which is called the power angle characteristics as illustrated in Figure 5b (blue sinusoidal curve).

Referring to Figure 21, and by taking the first and second derivative of eq. (7), the following is obtained:

$$\begin{aligned} \dot{\theta} &= \omega_m + \dot{\delta}, \text{ and } \dot{\delta} = \omega - \omega_m \\ \ddot{\theta} &= \ddot{\delta} \end{aligned} \quad (11)$$

where $\omega_m = \omega_{ref}$ is the nominal speed, and $\ddot{\theta}$ is the derivative of system frequency.

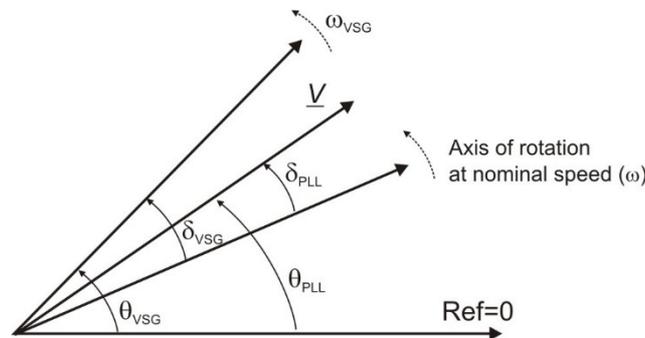


Figure 21. Phasor diagram of SG rotation and angles (synchronous rotation frame)

In Figure 21, θ_{VSG} , δ_{VSG} , ω_{ref} , θ_{PLL} , and δ_{PLL} are the angular position of VSG, transformation (phase) angle of VSG, grid nominal speed (frequency), phase angle and speed deviation of PLL. V is the measured voltage.